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Sciences**

**Doctoral School of Educational Sciences,  
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**Summary of Doctoral (PhD) Thesis**

**The realisation and critical points of the competence basic  
mathematics education in the teacher training field**

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# 1. Introduction

## **The reason of the topic choosing and the existing questions**

The present changes and problems of the public education make the examination with this topic justified. In every country of the world there are researches for the mathematics' modernisation and improvement of its efficiency on different levels of the education, among others in the higher education. One of these reform directions is the comparative education-methodological research (Ballér, 1990, Bábosik and Kárpáti, 2002), while the other branch is the learning research. Because of the changed learning habits the exact examination of this field has become urgent. My dissertation would like to contribute to these aims with the examination of the mathematics education in the teacher training field. Because of the changes nowadays we had to think over again the construction and structure of the teacher training and the syllabus for teaching, too. On the basis of the new Educational and Outgoing Requirements, in the teacher training institutions working in Hungary the revision of the sample syllabusses had to be done by the beginning of the school-year 2017. During the reforming the learning material the number of the lessons decreased. The different higher educational institutions divided differently the compulsory credit and lessons' quantity, therefore the taught material has changed everywhere, too.

In my dissertation my main aim is the examination of the revision's effects and of the changes in the mathematical knowledge of the students. I undertake to work out three main fields. The first part is to examine how certain institutions changed their sample syllabusses following the new Educational and Outgoing Requirements. The second field searches whether with the help of the separated subject pedagogical courses in the teaching plans or with the help of the methodology of the founding subjects could the teacher training students prepare better for their later profession. The third pillar of the dissertation is the examination of the mathematical knowledge of the students who take part in the basic teacher training: if their mathematical skills and abilities are suitable to the requirements for the mathematical competence.

## 2. Literary background

### 2.1. The mathematical competence

The teacher training students must know and appreciate the features of the mathematical competences to make good use of their knowledge during the education. With its help they can improve their own practical knowledge, but during teaching the children they have to take this in consideration this, too. One of the most determining organisations of the present social and economical processes is the OECD, that is why we describe that definition here which is connected to the PISA surveys organised by them: „the mathematical competence is such a preparation that makes us suitable to the identification, understanding, handling of the mathematical problems, furthermore to form a well-founded opinion about the role of the mathematics in the individual’s present and futural professional career, forming private life, family and social contacts ” (PISA, 2000)

On the basis of the determination the following components of the competence have been determined (Niss, 2000):

- mathematical thinking, conclusion
- mathematical argumentation, verification
- mathematical communication
- mathematical modelling
- raising problem and problem solution
- representation
- symbolical and formal language and operations
- using tools

Competence fields to form and improve - during teaching mathematics of the teacher training faculty, which have to be at students’ disposal during the education (Czeglédy, 2010:3):

- I. Understandable reading, text analysing.
- II. Ability for problem solving.
- III. Counting skills.
- IV. Expertise in thinking operations.
- V. Creative personal features.

- VI. Skills for algorithmic thinking.
- VII. Ability to plan solutions, systematics, expediency.
- VIII. Combinatorial thinking.
- IX. Ability for practical application.
- X. Functional way of thinking.
- XI. Orientation in time and space.
- XII. Demand of proving, power of judgment.
- XIII. Recognition and application of geometrical transformations in practice.
- XIV. Application of probability calculation theory in everyday life.

To the measuring of the above mentioned science branches are connected also international examinations, in which the examination of the students' mathematical knowledge, ability, skills are built around three directing principles:

- to what kind of mathematical content can the given problems and questions be connected;
- through what kind of competences' mobilization can be formed the examined real problem to a mathematical problem and can be solved after it;
- in what kind of situations and contexts can the tasks appear.

We also followed this division during the analysis of the tests after abilities.

## **2.2. Changes in syllabus, the examination of the teaching plan's net of the teacher training institutions**

Because of our examinations after the analysis of the mathematical competences we have to mention the changes in syllabus, because every change in the teacher training occurred following the introduction of the Educational and Outgoing Requirements. The institutions dealing with teacher training revised and reformed after different conceptions their syllabus from the school-year 2017/2018 to suit the new Educational and Outgoing Requirements. The aim was the establishment of the continuously raising educational culture which suits the maintainable, real economical demands.

To the achievement of the quality teacher training, which suits the changing social demands, the education of mathematics has to contribute, too. The graduated students at the faculty have to be prepared for the complex improvement of their students.

In the following charts we collected the most Hungarian institutions that deal with teacher training to show how they have reformed their educational system compared to their old syllabus in the field of mathematics.

It is a positive fact that following the transformation some subjects became coordinated enough, there is no overlapping in them, so the institutions are able to establish the complex education that are described in the requirements. Integrated knowledge can be established with them, skills and abilities can be improved by them.

What we must mention as a negativism is that as a consequence of the Educational and Outgoing Requirements in the most places it came to the reduction of the lessons. However this reduced the overloading of the students, but it has no positive effect on the establishment of the subjectal and methodological knowledge. The establishment of contacts and the personal contact give the essence of teacher training. Through subjects' pedagogies teacher training students can get to know the methods and tools, while a kind of attitude establishes in them that will be necessary in their whole life to perform their profession. We see the danger of fewer contact lessons that if suitable learning habits of students are not established then they can cope harder or no way with their own tasks.

The majority of the institutions decided that they reduce the number of disciplinar founding subjects to make stronger the rate of the subject pedagogy. Many times the founding courses are not useful considerably, the futural teachers can not find the contact points with their later profession. The courses of subject pedagogy help them also in this sense to prepare for their later work.

During the examination of syllabusses can be seen that there are such institutions (Eger, Sáropatak) where originally a little subject pedagogy was built in the subjects. In this places there was stress on the professional subjects. Referring to the personal conversations with academic teachers and as the results of our surveys – described later – it can be stated that in this frame system the instructors can not establish the imagination that instead of subject pedagogy every professional and mathematical subject should be also methodological subject. During preparing the dissertation we ourselves suggested the revision of their syllabus for the above mentioned institutions on the basis of the results of previous examinations. I mean a big result that on the basis of our suggestions the independent subject pedagogical course – seperated from the professional subjects - has been introduced into the new model syllabus.

1. chart 2015 Model syllabus

Town	I.		II.		III.		IV.		V.		VI.		VII.		VIII.
	Maths theory I.	2+2	Maths theory II	2+2	Subject-pedagogy	1+2	Subject-pedagogy	1+1					Elementary maths	0+2	
Baja	Maths theory I.	2+2	Maths theory II	2+2	Subject-pedagogy	1+2	Subject-pedagogy	1+1					Elementary maths	0+2	
Budapest	Introductory maths	0+2	Founding of maths concepts	0+2	Subject-pedagogy	0+2	Subject-pedagogy	0+2	Geometry	0+2	Kombinatory, probability Maths concept basic II	0+2 1+0			
Debrecen	Maths I	2+2	Maths II	2+2	Subject-pedagogy	2+0	Subject-pedagogy	0+2	Subject-pedagogy	0+2					
Győr	Maths I	1+2	Maths II	0+2	Subject-pedagogy	2+1	Maths III	1+1					Elementary maths	0+1	
Károli-Bp.	Maths I	0+2	Maths II	2+0	Subject-pedagogy	0+2	Subject-pedagogy	2+0	Subject-pedagogy	2+0	Maths III	0+2			
Kecskemét	Maths I	2+2	Maths II	2+2	Subject-pedagogy	0+2	Subject-pedagogy	0+2			Subject-pedagogy	0+2			
Nagykőrös	Maths I	0+2	Maths II	2+0	Subject-pedagogy	0+2	Subject-pedagogy	2+0	Subject-pedagogy	2+0	Maths III	0+2			
Nyíregyháza	Maths I	2+2	Maths II	1+2	Subject-pedagogy	2+1	Subject-pedagogy	1+1			Elementary maths	0+3			
Sárospatak-Eger	Thinking methods	0+2	Number theory	0+2	Geometry	2+0	Function science	0+2	Statistics	2+0	Science history and Maths curiosities	0+2			
Szarvas			Maths founding	1+2	Methodology of maths knowledge	0+3	Subject-pedagogy	1+2	Maths problem solving and improving abilities	1+2					
Szeged	Maths practice Maths I	2+2 0+2	Maths II Subject-pedagogy	2+1 2+1	Subject-pedagogy	0+2	Subject-pedagogy	0+2	Elementary maths	0+1	Elementary maths	0+1			
Széksárd	Maths I	2+0	Maths II	0+2	Subject-pedagogy	2+0	Subject-pedagogy	0+2							
Vác	Maths I	2+2	Maths II	2+2	Subject-pedagogy	0+4	Subject-pedagogy	1+2							

2. chart 2017 Syllabus

Town	I		II		III		IV		V		VI		VII	
	Maths I	2+2	Maths II	2+2	Subject-pedagogy I	1+2	Subject-pedagogy	1+1	Geometry and subject pedagogy	0+2	Combinatory, probability thinking improvement with plays	0+2	Elementary maths	0+2
Buda	Maths practice Basis of maths concepts I	2+2	Basis of Maths concepts II	0+2	Teaching of counting, number figures-expanding	0+2	Thinking methods	0+2	Geometry and subject pedagogy	0+2	Combinatory, probability thinking improvement with plays	0+2	Elementary maths	0+2
Budapest	Maths practice Basis of maths concepts I	2+2	Basis of Maths concepts II	0+2	Teaching of counting, number figures-expanding	0+2	Thinking methods	0+2	Geometry and subject pedagogy	0+2	Combinatory, probability thinking improvement with plays	0+2	Elementary maths	0+2
Debrecen	Mat I	1+2	Maths II	1+2	Subject-pedagogy	0+2	Subject-pedagogy	0+2	Subject-pedagogy	0+2	Subject-pedagogy	0+2	Elementary maths	0+2
Győr	Subject-pedagogy	1+2	Subject-pedagogy	0+2	Subject-pedagogy	2+1	Subject-pedagogy	0+2	Subject-pedagogy	0+2	Subject-pedagogy	0+2	Elementary maths	0+2
Károli-Bp.			Maths basic knowledge Subject-pedagogy	2+0 0+2	Subject-pedagogy	0+2	Subject-pedagogy	0+2	Subject-pedagogy	0+2	Subject-pedagogy	0+2	Subject-pedagogy	2+0
Kecskemét	Maths I	2+2	Maths II	2+2	Subject-pedagogy	0+2	Subject-pedagogy	0+2	Subject-pedagogy	0+2	Subject-pedagogy	0+2	Subject-pedagogy	2+0
Nagykőrös			Basic knowledge Subject-pedagogy	2+0 0+2	Subject-pedagogy	0+2	Subject-pedagogy	2+0	Subject-pedagogy	0+2	Subject-pedagogy	0+2	Subject-pedagogy	2+0
Nyíregyháza	Maths I	1+2	Maths II	1+1	Subject-pedagogy	1+2	Subject-pedagogy	0+2	Subject-pedagogy	0+2	Subject-pedagogy	0+2	Subject-pedagogy	2+0
Eger, Jász, Sárospatak	Thinking methods	0+2	Number theory	0+2	Subject-pedagogy	2+0	Subject-pedagogy	0+2	Subject-pedagogy	0+2	Subject-pedagogy	0+2	Subject-pedagogy	2+0
Szarvas			Maths founding	1+2	Methodology of Maths knowledge	0+3	Subject-pedagogy	1+2	Maths problem solving	1+2	Subject-pedagogy	0+2	Subject-pedagogy	2+0
Szeged	Maths practice Maths I	0+2 0+3	Maths II Subject-pedagogy	1+1 2+0	Subject-pedagogy		Subject-pedagogy	0+3	Subject-pedagogy	0+3	Subject-pedagogy	0+2	Subject-pedagogy	2+0
Székelyvár	Maths I	2+0	Maths II	0+2 sz2	Subject-pedagogy	sz 2	Subject-pedagogy	0+2	Subject-pedagogy	0+2	Subject-pedagogy	0+2	Subject-pedagogy	2+0
Szombathely	Maths practice Maths concepts I	2+2	Basis of Maths concepts II	0+2	Teaching of counting, expanding of number figures	0+2	Thinking methods	0+2	Geometry and subject pedagogy	0+2	Improvement of combinatorial probability thinking	0+2	Subject-pedagogy	2+0
Vác	Maths I	2+2	Maths II	1+2	Subject-pedagogy	0+3	Subject-pedagogy	1+2	Subject-pedagogy	1+2	Subject-pedagogy	0+2	Subject-pedagogy	1+1

On the basis of the two charts we prepared a summarized chart to the favour of the better transparency. In the chart underneath the syllabus lessons can be seen before and after the transformation of the teaching plan, divided into theoretical, practical and total entire lessons' number. While in the previous charts the change of the subjects' titles and features can be seen, in the chart underneath the reduction of the lessons is shown significantly enough. In different institutions, that deal with teacher training, unfortunately the various number of the differences is thanks to the fact, what kind of place the mathematics teaching study department takes place in the hierarchy of the given university and college departments. The practice oriented education can give the new learning central educational culture and the instructors of the institutions know this exactly, too. The frontal taught study material has less practical use than if the graduate student can try his/her own knowledge, eventually there are some possibility for microteaching. But that fact is very unfortunate, that even so in the most places it came to the reduction of the total lessons' number. According to the number, one or two lessons in minus cannot be seen much, but if we see that in the higher education the lesson number of the subjects also moves between one-two lessons per half-year, then we can see that with these reductions of the lesson number whole subjects can disappear and have already disappeared, too.

3. chart – Summary of the lesson numbers' changing

Institution	Lecture		difference	Practice		difference	summarized		difference
	2015 syllabus	2017 syllabus		2015 syllabus	2017 syllabus		2015 syllabus	2017 syllabus	
Baja	6	6	<b>0</b>	9	9	<b>0</b>	15	15	<b>0</b>
Budapest	1	0	<b>-1</b>	12	12	<b>0</b>	13	12	<b>-1</b>
Debrecen	6	2	<b>-4</b>	8	10	<b>+2</b>	14	12	<b>-2</b>
Győr	4	3	<b>-1</b>	7	7	<b>0</b>	11	10	<b>-1</b>
Károli-Bp.	6	4	<b>-2</b>	6	6	<b>0</b>	12	10	<b>-2</b>
Kecskemét	4	4	<b>0</b>	10	10	<b>0</b>	14	14	<b>0</b>
Nagykőrös	6	6	<b>0</b>	6	4	<b>-2</b>	12	10	<b>-2</b>
Nyíregyháza	6	3	<b>-3</b>	9	7	<b>-2</b>	15	10	<b>-5</b>
Sárospatak-Eger	4	6	<b>2</b>	8	8	<b>0</b>	12	14	<b>+2</b>
Szarvas	3	3	<b>0</b>	9	9	<b>0</b>	12	12	<b>0</b>
Szeged	4	3	<b>-1</b>	10	12	<b>+2</b>	14+2 free choice	15	<b>-1</b>
Szekszárd	4	2	<b>-2</b>	4	4	<b>0</b>	8	6+4 free choice	<b>-2</b>
Vác	5	6	<b>1</b>	10	10	<b>0</b>	15	16	<b>+1</b>



### 3. The research

After the comparison of the syllabuses in the second part of our examination we examined the mathematics knowledge (special subject and special methodology) of the students. Within this we laid the stress on the abilities because in the learning and teaching of mathematics those abilities are the most important that are needed to the learning and the acquisition of the knowledge.

Between 2014 and 2017 we made overall, longitudinal and cross-section examination about the mathematical knowledge of the teacher training students at the beginning and at the end of the methodological course. The pre-surveys were going in the school-years of 2014/2015 and 2015/2016. Summarized the experiences of the pre-surveys we made a test included 27 tasks and an equivalent test version. This was needed because we made the examination in more Hungarian teacher training institutions at the beginning and at the end of the subject pedagogy course. In those training institutions where there are no founding and subjectpedagogical subjects, there we got the test written at the beginning of the mathematical courses and at the end of the last such course. The examination was a survey with more groups, with pre and final tests. In the interests of the representativity we worked hard to cover the places of the Hungarian teacher training. Within big universities, „big-cities” institutions and rural „small-town” institutions we made the examination, too. To help to verify our hypothesis we divided the groups during the assessment according to its place in the subjectpedagogy syllabus and the quantity of the subjects. Those colleges, universities belonged to one of the groups in those the subjectpedagogy is not separated from the founding subjects. Those institutions formed the other group where there is a subjectpedagogical course for half-year following the founding subjects and in the third group came the ones where the teacher training students study for 2 semesters the secrets of mathematics teaching.

We managed to make a survey in the group of college leavers graduates, furthermore we got a questionnaire made by them. Students in their fourth semester can already see the structure of the education with experience, they could experience which were those parts that were useful for them during their studies.

### **3.1 Hypothesis**

Our hypotheses relate to the examination of the special subject's and methodological knowledge in the mirror of different types' syllabuses.

That is why we stated the following hypotheses:

H<sub>1</sub>: The beginner teacher students miss the exact knowledge in connection with mathematical concepts.

H<sub>2</sub>: The special subject's knowledge of the undergraduates - who absolve the subjectpedagogical course - improve more opposite those undergraduates who absolve only the founding subjects.

H<sub>3</sub>: The special methodological knowledge improves significantly in case of the undergraduates who absolve the methodological course opposite those undergraduates who do not absolve a separate methodological course.

H<sub>4</sub>: The college leavers graduates have the right outgoing competences.

### **3.2 The methods and fields of the research**

#### **3.2.1 Previous examinations**

In the school-year of 2014/2015 we made examinations only on one group to make sure the existence of our hypotheses. This survey was done at the Széchenyi University Apáczai Csere János Faculty, 56 undergraduates took part in it. To write the test the undergraduates had 90 minutes in every case. In autumn of 2015 the first control-group's measure happened where we examined the students of the Teacher Training College in Győr (54 people) and in Eger, earlier still named Eszterházy Károly College (16 people). At the beginning and end of the subjectpedagogical course we measured the undergraduates, then we made the writing of the test at the institution that assured the control group at the beginning and after closing of their mathematics courses.

#### **3.2.2 Big sample measure**

The test for the big sample measure did not contain any knowledge and abilities over the mathematical syllabus content of the lower years of primary school. During the examination we prepared the parallel version of the first mathematical test, too.

In this parallel test there were isomorph tasks compared to the pre-test, they were different only in their context (for example the used concrete numbers, the names of the people and objects as well as the wording of the problems and so on).

Shortly before the beginning of the school-year 2016/2017 we sent the test-sheets to those other institutions that took part. 174 people wrote the tests together, but we could assess 168 tests, because the rest 6 papers did not contain any valuable results. We gave concrete orders related to how the test must be made filled by the undergraduate candidates. These institutions were the teacher training institutions and faculties of Debrecen, Baja, Szeged, Sáropatak beside the institutions of Győr and Eger. The test was written in every institution in the first week of the school-year 2016/2017. The survey took 90 minutes. Each of them contained 27 tasks, on which you could get 1, 2, 3, 4 or 5 points. In most cases a certain number gave the answer. In other cases a working out and a regular description of the tasks were needed. The organisation and arrangement of the after-test happened in the same way like this of the pre-test.

### 3.2.3 The examination of the tasks according to abilities

We ranked the tasks into six types, which were born by mixing a given content and a cognitive dimension. The dimension of the content determined the two categories: arithmetic and geometric ones. The cognitive dimension resulted three categories: declarative knowledges, procedural knowledges, strategies and problem-solving abilities. The declarative knowledges mean the routine-type tasks and their execution with the application of definitions. The procedural skills are already more complex but they still require the completion of standard-type tasks. The problemsolving abilities are present in those tasks where the reflective view and the generalization are needed, we have to handle complex problems to their solution. This division resulted six part-groups of the tasks. In the test we gave several examples to the certain part-groups.

### 3.2.4 The assessment of the college-leaver graduates

We also had the opportunity to examine those graduates who had already absolved their mathematical studies and stand before the beginning of their practice. We examined whether they own those mathematical knowledge that is needed to their profession.

## 4. The result of the empirical examination

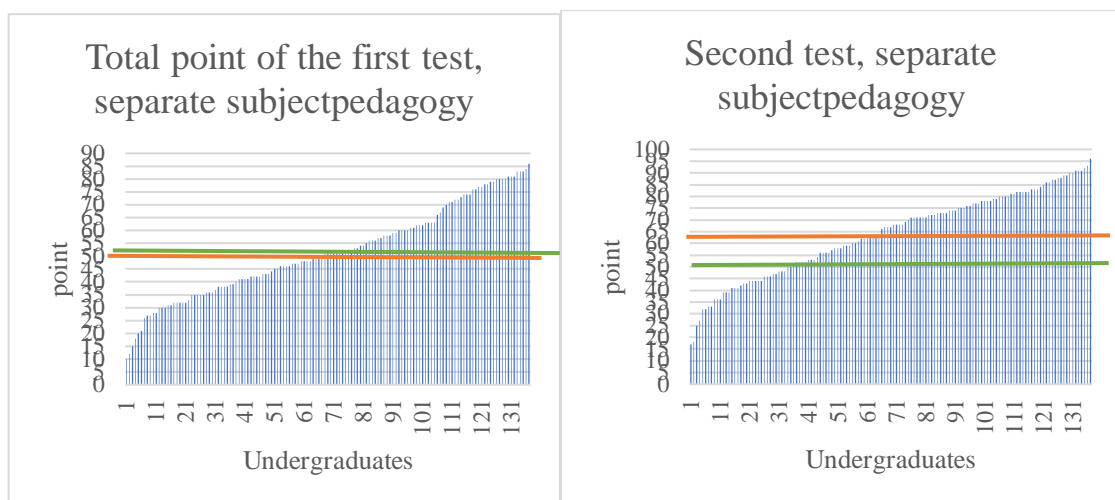
### 4.1 The summarized results of the big-sample's examination

In short we sum up the got results, we go on detailed only on the big-sample' assessment. The next charts and graphs contain summarized the results of the tests.

4. chart Result of the big-sample's examination

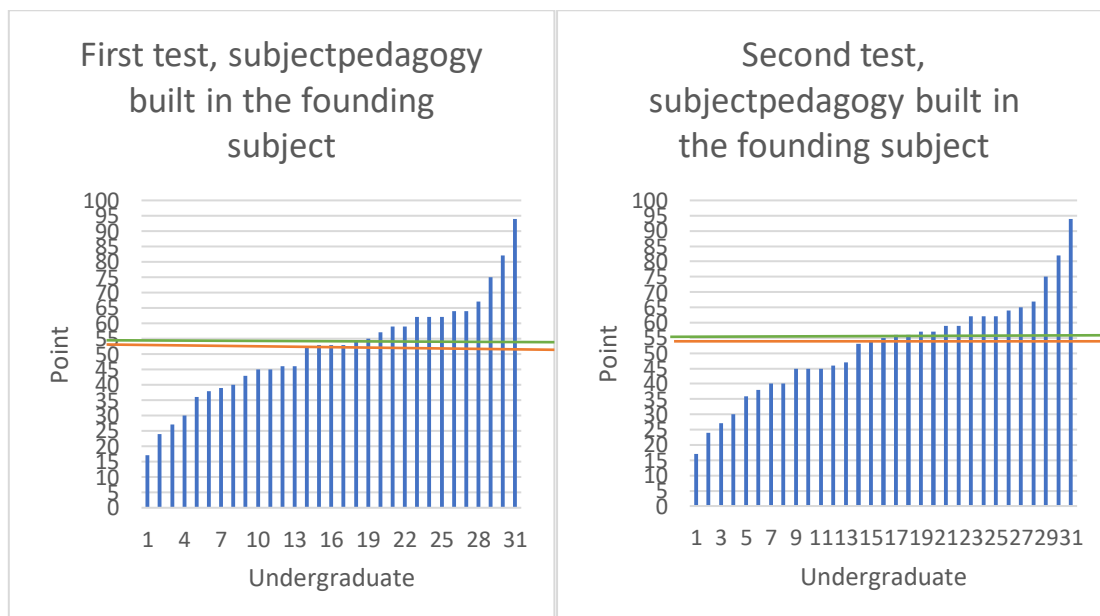
Subjectpedagogy as a seperated subject							Methodology built in the founding subject						
beginning of the year (point)			end of the year (point)			element nr.sample	beginning of the year (point)			end of the year (point)			element nr.sample
average	dispersion	median	average	dispersion	median		average	dispersion	median	average	dispersion	median	
51,7	17,8	51	64,0	18,0	67	137	51,7	16,1	53	52,2	16,6	55	31

Summarized the results of undergraduates who study subjectpedagogy separated from the founding subject we can see that they reached 51,7 points on average (dispersion 17,8 points) at the beginning of the half-year, while at the end of the half-year the result became 64,0 points (dispersion 18,0), what means an improvement of 23%. We put the individual results into one graphs. We also indicated the median (green line) and the 50% result (53 points, yellow line). With this we tried to show that the half of the students do not reach the 50% which is needed to the satisfactory performance. By the end of the semester more than the half of the undergraduates would already perform the subject (70% of the undergraduates), if this test would be the requirement.



1. diagram The results of the undergraduates who study separate subjectpedagogy, first and second test

We examined separately those institutions where at the beginning of the examination there were not any separate subjectpedagogical course, only some methodology was educated built in the founding subjects, supposing that the qualification of the actual teacher made this possible. We have to mark it especially – for example in analysis – because it is not sure that in a given topic excellent teacher, so he/she can connect the founding disciplinar subject and the methodology in a more difficult way in his/her lessons. So the quantity of the methodology depends on the field of knowledge and research of the given instructor, in spite of the courses that are kept by the prepared, expertised colleagues who teach separate methodology. Our examination showed that the mathematical knowledge of the undergraduates, who study on the base of such a syllabus, does not change significantly. We are going to explain this detailed in the coming chapters, here we only show the total result. In case of the test - written at the beginning of the studies – can be seen that the median coincides with the performance of 50%. Opposite the results of the institutions, who teach also separate subjectpedagogy, at these institutions any big change is experienced in the performance during the second test. The tasks' solving success does not improve, the meanings of the special words do not develop, neither the learning of the methodological expressions, nor the notice making of the text based maths exercises happen in a right way.



2. diagram The results of the undergraduates who study methodology built in the founding subject, first and second test

After the statement of the overall view we also examined the change of the tasks' solving success, but we made this known only in case of the institutions that teach separate subject-pedagogy, because in other institutions there was a minimal difference in the results of the two tests.

Beside the observation of the improvement the analyses are also suitable to examine which abilities' improvement is shown at least during the tasks' solving. It can be stated that the biggest weaknesses, which need further improvement, appeared in the following abilities:

- It is difficult for the undergraduates to explain such tasks that need to give reasons.
- They mean the preparation of the plan as an unuseful, timewasting activity.
- We experienced during the use of the marks, that the replacement in formulas is difficult, they substitute the dates wrong from the blackboard or they use the formulas in a bad way.
- Furthermore we experienced at the text-based maths tasks that during the planned solution problems came up, too. The solution was in many cases only the wrong copy of a learnt example.
- In the text-based geometry task, the undergraduates did not manage the preparation of the relevant diagram, either.
- While in case of some systematics requiring tasks, the research of all right solutions happened due to bad application of systematization.
- We meant the lack of the effort to the whole solution for their undermotivation, lacks in the knowledge material and for their study without any mind.

From our measure it can be seen that we must teach emphasized the solution's and representation's methods of the text-based tasks for our teacher-candidates.

The methodological courses help the undergraduates with the improvement of comprehension and interpretation and with the representation of the problem. In the frame of the subject-pedagogical course they get knowledge about the methodological structure of mathematics, the forming possibilities of numberconcept, operation-concept in the solution of text-based problems with lower elementary methods, that are indispensable referring to the above mention facts.

## 4.2 The results of the comparative examination

We compared the written tests at the beginning and end of the semester among the institutions that teach subject-pedagogy separately and integrate methodology in the founding subjects.

On the base of our previous examination our supposition was that who study subject-pedagogy in the frame of a separate subject, their results will be better, the concepts will be exacter, their system will be uniform. So we examined the results on base of more views. Firstly we saw separately whether we reached the required improvement in the single institutions. We tested the performance change of the different institutions with the one-sample probe, because we wanted to see whether a significant difference can be shown between the performances before and after the subject-pedagogical courses. The institutions are marked in the diagram by letter-charakter, we do not make known the names of the institutions referring to the law of different institutions and students.

5. chart The result of the big-sample measure in all the examined institutions

Institution	Elementnumber of the sample	First test		Second test	
		average	dispersion	average	dispersion
A	43	53,6	17,7	66,7	15,2
B	32	57,3	17,8	61,2	18,2
C	52	46,1	17,2	62,4	20,6
D	10	55,5	20,8	69,5	12,3
E	16	50,2	18,5	53,6	15,9
F	15	51,2	16,6	52,2	16,6

The result of the comparison of the pre- and final examination's results average refers that the mathematical methodology had a meaningful and favourable effect on the mathematical competence of the students. We verified our hypotesisses: the little knowledge can be expanded, improved, the special methodological knowledge can be formed in an effective way.

6. chart The result of the big-sample measure in all institutions (on the significance level of 5%)

Institution	value of the t-probe	freedom degree	$t_{\text{chart}}$	result
A	7,31	42	1,684	significant divergence
B	1,86	31	1,697	significant divergence
C	8,26	51	1,676	significant divergence
D	4,58	9	1,812	significant divergence
E	0,47	15	1,753	no divergence
F	0,67	14	1,761	no divergence

When we put the institutions on the scale from that point of view whether the subject-pedagogy is as a separate course in the sample-syllabus we realised that in those institutions where the founding subjects contain the subject-pedagogy, no change happens. It is teacher-depending how much he/she teaches the basic knowledge in the own subject and our experience shows that the students need to repeat again their elementary school's lower-years knowledge in the frame of a separate subject. This kind of our examination summed up that the methodology of mathematics has to exist as a distinct subject to improve the special subject's and special methodological knowledge in a suitable degree.

#### 4.3. The examination of the tasks due to skills

Inside the ability-improvement adjusted to the subjects we distinguish two groups. The aims of one group stay in the frame of subject teaching, while those belong to the other group in that the abilities'circle for improvement overshadow the usual content of the subject. From the beginning the thinking-improvement belongs to the mathematics.

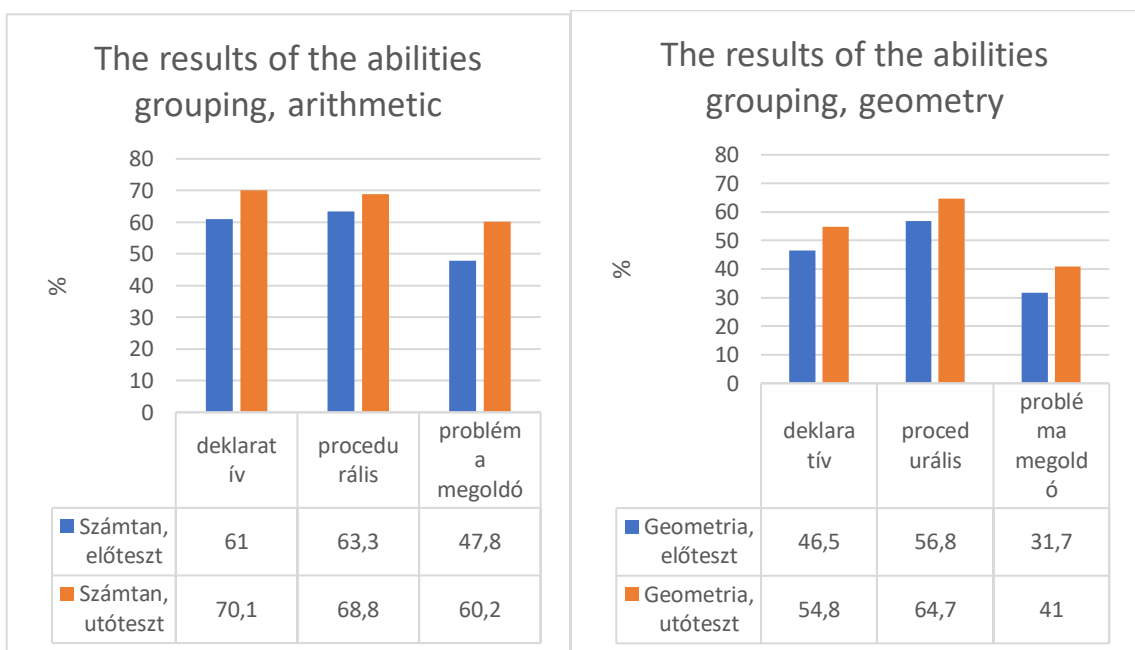
We examined for supporting all this, whether the improvement of the tasks due to types came true and if yes then in how much degree. In the diagrams the percental results are shown because the maximal points of the individual tasks were other, so the comparison could not be done.

Although the tasks of the test do not cross over the frames of the elementary school lower years' syllabus, there are still such elements that do not reach the 50% of the maximal points. The worst results were born in the topic of geometry. Its cause can be, that the



elementary geometry gets in the background in the education. If we add the problemsolving category to that, we can see that the point of intersection of these gives the worst result. Regarding the three cognitive dimensions the text-based, problem-solving requiring tasks cause the most difficulties for the undergraduates. As a consequence it can be stated that in case of the learning-material improvement the stress must be put on the problemsolving thinking and comprehension.

For the favour of the comparability we showed the results on a graph – looking separately the dimensions of the arithmetic and geometry –, this way the change of the part-dimensions distincts much better.



3. diagram The grouping of abilities

Usually the representation of the problem causes a bigger difficulty than the execution of the solving process. To get to the solution and the aim-state from the beginning state of the task, the intermediate steps must be right throughout. In many cases the translation of the problem in mathematical language is more difficult, the completion of the after needed and allowed operators already do not cause such a big problem.

#### 4.4 The assessment of the finalist graduates

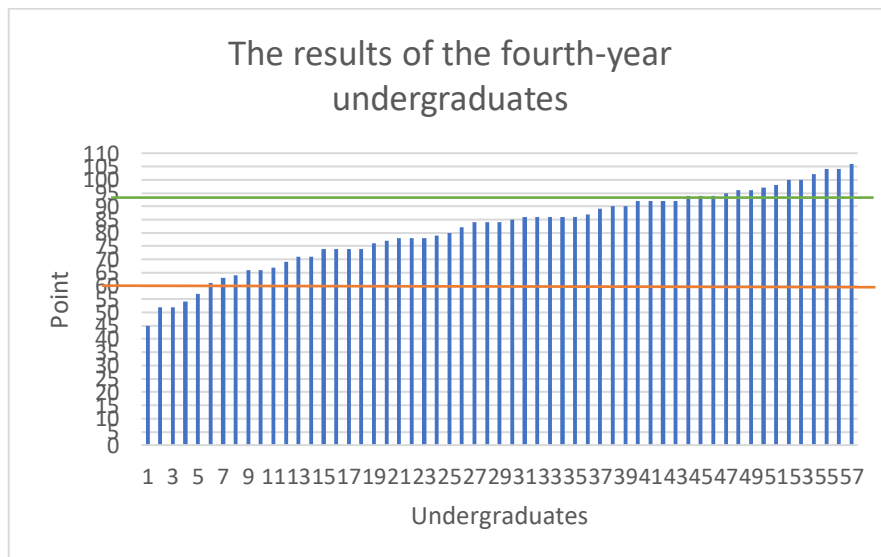
It is an important question to examine whether we manage to prepare the undergraduates to their futural profession. We also wanted to get an answer to our question whether the

undergraduates can reach the needed mathematical competences to their work-start till the end of the training. That is why we got the test written also by the undergraduates who had already absolved all mathematical courses. We got the following results: we got the test written by 57 people and as the graph also shows, it managed much better than for the undergraduates who are in the middle of their studies. The average of the test is 80,2 points (dispersion: 18 points), its median 84 points. In the chart the differences can be seen compared with the results of the second school-year.

7. chart The results of the second and fourth year

School-year	Element-number	Average (point)	Dispersion
2.	137	64,1	18,1
4.	58	80,2	18,0

The 94,7% of the undergraduates reached the 53 points needed to the 50%. So we can say that they are prepared for their profession from the point of view of mathematical knowledge. The median is also much higher (84 points) than in the second school-year. We can see the summarized results of the fourth-year undergraduates in the 20. diagram.



4. diagram The result of the fourth-year undergraduates

But it is true also here that the undergraduates perform the weakest result in the tasks that contain methodological knowledge, are complex or text-based. In case of these tasks the undevelopment of the problemsolving thinking caused the most difficulties, whose elements are the ability of the comprehensive reading, the finding of the relations, the planning, the execution of the plan and the reason giving of the result.

Beside the same dispersion a significant divergence can be experienced in the knowledge of the two school-years. The experiences in the practices helped the fourth-year undergraduates to change of their attitude to the mathematics. As a reflection to their earlier studies and performance, consequently they gave a better performance. Several experts who deal with education-science (for example Tillema) lay the stress on the reflective practice at the obtaining and deeping of the knowledge. It is important that the undergraduates already filter that consequence from their earlier experiences that they need the exact mathematical knowledge.

#### **4.5 The results of the questionnaire survey**

Although the examination of the questionnaire survey can be less exact, from the answers to our questions in connection with mathematical competences it can be seen that the improvement of the logical consequences is held the most important, almost 21 people mentioned the thinking abilities. Beside this the improvement of space-seeing, counting skill, rule-recognising follow in order of importance. Everybody thinks that the education of the logical thinking, the improvement of the concentration must be realised through the improvement of mathematical competences, too.

At the undergraduates the secure methodological knowledge enjoys priority among the competent teacher criteria, which is supplemented with a helpful, patient teacher attitude. At our question to the reinforcement of the pedagogical competences and order of importance, the personality improvement and the needed methodological preparation of the undergraduates to an inclusive education erect. This is in one line with the above already analysed point of view, therefore the finalist undergraduates consider important the suitable methodological knowledge storage. The support of the learning also appears in the first half of the list. The control of the classroom processes is the last in the order, this is characteristic to the beginner pedagogists, because they try to realise even more the plan of the lesson material.

The third part is about the got experiences during their studies. How they find the high-educational studies, which subjects helped them during their later practices. They reported uniformly that they felt the subject-pedagogical course the most determined in the preparation for their futural career. They felt the learning of the methods very significant, as well as they could get to know the treatment with demonstrative tools and

their application possibilities in the educational process. They felt much the syllabus of the founding subjects and they could not find the relations among the materials that they must teach later.

We also asked the undergraduates during the filling in the questionnaire to declare whether their attitude to the mathematics changed during the training. Furthermore what they still expected or what they would change in the training. To these questions such answers arrived which were in harmony with the previous statements. The practical subjects, which prepare for the futural profession, helped the undergraduates to change their attitude to the mathematics. Opposite the syllabus reforms the pedagogist candidates would have a demand to further practical lessons. They would also have considered good even if they would be „forced” to study further mathematical knowledge by doing more tests and exams. We can conclude from this, that although the incoming mathematical knowledge of the students is little and missing, they have the demand of the improvement, if they get the suitable motivation and we form their view, too. We try to modify the structure, syllabus of our subjects year by year keeping before our eyes these kinds of information and results.

#### **4.6. The summary of the results**

From the mathematical subject - still loved in the age of a little pupil – becomes a less and less popular studium in higher classes. The knowledge of the participants in the teaching-learning process is determined by their attitude to the mathematics in many cases, they even more lose their interest in mathematics and the subject produces a bigger and bigger anxiety in them. The solution can be if we establish such an education which can help to change their wrong professional knowledge and damaged attitude, parallel with this their attitude both to the subject and its teaching should change into a positive direction. The task of the pedagogist training higher institutions is to prepare the pedagogists’ candidates to be able to face the challenges of their later career, to be able to the self-training as well as to be able to prevent the change of the attitude above or to keep it on a level as low as possible with the help of their methodological preparation.

The results of the experiments and questionnaires during our present research show for us that the separate subject-pedagogical courses are needed in the syllabus of the teacher training institutions. Most of the institutions also thought this when the syllabus

reforms were done. The results of almost 300 undergraduates altogether, who take part in the examination, show that the needed improvement can be reached in the mathematical subject and special methodological knowledge.

On the basis of the analysing assessment of the research we sum up our determinations in connection with the hypotheses of the examination as follows. Our hypotheses were real, because the drawing up in those reflects in the reached results:

- The first-year undergraduates miss the exact knowledges of the mathematical concepts: it was proved, that with the occasion of the mathematical courses, enough time is also needed to the replacement of the missing knowledge, to the correction of the wrong knowledges, the teaching and mediation of the learning material in the syllabuses are not enough.
- The subject knowledge of the undergraduates, who absolve the methodological course, improves by the end of the course: our examinations show how the such type of knowledge of the undergraduates can be improved.
- One of the most important statement of our examination is that in the teacher training from the present known two-types approaching (where one works with the separate special methodological courses, the other one inserts the special methodological knowledge in the special subjects), the level of the special methodological knowledge is significantly higher than after the separate methodological course. Those undergraduates solved the tasks including methodological items significantly better at the end of the semester, who listen to the subject-pedagogical course separately from the founding course. The previous undergraduates made with more success the regular description of the text-based tasks or the completion of the tasks with the given methods.
- The finalist undergraduates own the suitable outgoing competences: They managed to study in a right way the knowledge needed to the mathematics teaching and to get the competences.

## **5. Future view, the application possibilities of the results**

To the complex improvement of the undergraduates the gradual structure, deeping and expanding of the mathematical contents are needed. The futural teachers must fully appreciate the application possibilities of mathematics in other subjects, its utility and practical utilization, helping with this to improve the key-competences.

Our research proved, that still in case of teacher training students in higher education, the teaching of mathematics must start from such phenomena which are well-known, interesting and have a motivating effect. Through this we can help with the learning processes.

That is why we worked hard to establish new methods on the base of the experiences of our examinations' results. Making and analysing lesson-plans belong to these methods – after the informing description of the syllabus-indicating tasks. Furthermore with help of videos we demonstrate the lessons' processes through lessons' analysing so that the undergraduates can have a sight into their later work's stages and tasks still before the practice. With help of micro-teachings we also reforce the reflective practice.

Further researches are needed to make clear what kind of a role the training programme of the new system plays in the basic mathematical competences' improvement of the teacher training students. This present test can establish such a tool beside the documentation of the improvement of the mathematical résumé knowledges and skills, with that the mathematical knowledge of the teacher training students can be improved useful. The materials, that are established during the development of the syllabus, also help to form the assessment system of the undergraduates (for example self-criticism). The whole test proved to be a valuable tool in the assessment of the undergraduates' entering-level and mathematical résumé knowledges. Finally in interest of the regular following and further improvement of the mathematical competence and methodological culture of the teacher training students it is needed to establish further auxiliary materials, tasks'banks – utilizing the possibilities of informational and communicational technologies – which can reforce the independent studying at home. At present we are planning such a project whose aim is to develop a computer programme, with its help the teacher training students can continuously assess and criticize their mathematical knowledges and skills on their own, too.

## Bibliography

- Bábosik, I., Kárpáti, A. (2002): *Összehasonlító Pedagógia, A nevelés és oktatás nemzetközi perspektívái*. BIP, Budapest
- Ballér, E. (1990): *Bevezetés a felsőoktatás didaktikájába*. Pedagógiai és pszichológiai szabad alternatív tárgyak útmutatói, Budapesti Közgazdaságtudományi Egyetem Pedagógiai Tanszék. Aula Kiadó, Budapest.
- Czeglédy, I. (2010): *Kompetenciaalapú matematikaoktatás*, TÁMOP Eger,
- Niss, Morgens. (2000): *Mathematical Competencies and the Learning of Mathematics: The Danish KOM Project*  
<http://www.math.chalmers.se/Math/Grundutb/CTH/mve375/1112/docs/KOMkompetenser.pdf> 2018. augusztus 12.
- Key Competencies. A developing concept in general compulsory education. Eurydice, 2002. European Commission. Directorate General for Education and Culture. Survey 5  
<https://docplayer.hu/3671914-Kulcskompetenciak-kulcskompetenciak.html>  
2018. augusztus 11.
- PISA, 2006  
[https://www.oktatas.hu/kozneveles/meresek/pisa/pisa\\_2006\\_meres](https://www.oktatas.hu/kozneveles/meresek/pisa/pisa_2006_meres) 2018. aug. 11

## Publication list

### Conference presentation:

#### Hungarian-language conference at Hungarian and foreign conferences (lectured):

Petz, T.: *Reflektív szemináriumok megvalósulásának lehetőségei a Matematika tantárgypedagógia tantárgy keretein belül.* Elmélet és gyakorlat a neveléstudományok és szakmódszertanok köréből. Štúrovo, Szlovákia, 2018.01.14.-15.

Petz, T.: *A matematikaoktatás változásai – hatása a tanítóképzésre/ Changes in teaching mathematics – their influence in teacher training.* Tudomány az oktatásért - oktatás a tudományért, Természettudományi Szekció, Nitra, 2015. 09. 17-18.

Petz, T.: *Tapasztalatok a tanítóképzésben való matematikaoktatás során.* Selye János Egyetem, "Innováció és kreativitás az oktatásban és a tudományban" Nemzetközi Tudományos Konferencia, Pedagógiai szekciók. (A kompetencia alapú oktatás elmélete és gyakorlata alszekció) Komárno, Szlovákia, 2015.09.16-17.

Petz, T.: *Tanító szakos hallgatók ismereteinek felmérése, hiányosságok fejlesztésének lehetőségei a kötelező oktatás keretein belül.* Matematikát, Fizikát és Informatikát Oktatók (MAFIOK) XXXIX. Országos Konferenciája, Kaposvár, 2015. 08. 24-26.

#### Foreign language presentation at an international conference (lectured):

Petz, T.: *Problem of the mathematics thinking: Mathematics knowledge of teacher training students.* Joint Austrian-Hungarian Mathematical Conference Győr, 2015.08.25-27. (A Bolyai János Matematikai Társulat és az Osztrák Matematikai Társaság közös szervezésében)

#### Hungarian-language conference at Hungarian and foreign conferences (not lectured):

Petz, T.: *Virtuális valóság a tanítóképzés matematika oktatásában.* X. Tantárgypedagógiai Konferencia, Baja, 2019. április

Petz, T.: *Szemléltetés és játék a tanító szakos hallgatók matematika óráin,* Rácz László Vándorgyűlés, Győr, 2018. július

Petz, T.: *Előzetes tudás vizsgálatának eredményei.* XX. Apáczai-napok Nemzetközi Tudományos Konferencia: "Semper Reformare". Konferencia helye, ideje: Győr, 2016. október

Petz, T.: *Tanítós hallgatók Dienes Zoltán nyomdokain,* 2. Dienes-nap, Matematikai Módszertani Konferencia, Eger, 2016. május



Petz, T.: *Előzetes tudás mérése*. XIX. Apáczai Napok Nemzetközi Konferencia: Gondolkodási struktúrák és kreativitás, Győr, 2015. október

### **Publications:**

Petz, T.: *Reflektív szemináriumok megvalósulásának lehetőségei a Matematika tantárgypedagógia tantárgy keretein belül*. In: J. Karlovitz (Eds.): *Elmélet és gyakorlat a neveléstudományok és szakmódszertanok köréből*. Komárno, International Research Institute, 2018. pp. 39-43.

Petz, T., Hoffman, M.: *The development of mathematical competences in Hungarian teacher training education*. ANNALES MATHEMATICAE ET INFORMATICAЕ 47, 2017. pp. 243-251.

Petz, T.: *Előzetes tudás vizsgálatának eredményei*. In: I. Lőrincz (Eds.): XX. Apáczai-napok Nemzetközi Tudományos Konferencia: "Semper Reformare". Győr, Széchenyi István Egyetem, 2017. pp. 242-248.

Petz, T.: *Előzetes tudás*. In: I. Lőrincz (Eds.): XIX. Apáczai-napok. Tudományos Konferencia. Tanulmánykötet: Gondolkodási struktúrák és kreativitás. Győr, 2016. pp. 321-328.

Petz, T.: *Problem of the mathematics thinking: Mathematics knowledge of teacher training students*. In: János Bolyai Mathematical Society, Austrian Mathematical Society (Eds.): *Joint Austrian-Hungarian Mathematical Conference 2015: Book of Abstracts*. Győr, pp. 18.

Petz, T.: *A matematikaoktatás és a valóság*. In: I. Lőrincz (Ed.): XVIII. Apáczai-napok. Tudományos Konferencia: *Quid est veritas? (Jn 18,38): Teóriák, hipotézisek és az igazság viszonya*. Nyugat-magyarországi Egyetem Kiadó, 2015. pp. 422-429.

Petz, T.: *How much more can a college's student than a primary school's student?: Mathematics knowledge of teacher training students* In: A. Komzsík, T. Szabó (Eds.): *Ab igne ignem: László Béla 75. születésnapjára: K 75. narodeninám Bélu Lászlóa*. Nitra: Univerzita Konštantína Filozofa v Nitre Fakulta Stredoeuropskych Studii, Europica varietas sorozat; 49. 2015. pp. 53-59.

Petz, T.: *Tapasztalatok a tanítóképzésben való matematikaoktatás során*. In: Gy. Juhász, Á. Nagy, T. Strédl, A. Tóth-Bakos (Eds.): *A Selye János Egyetem 2015-ös "Innováció és kreativitás az oktatásban és a tudományban" Nemzetközi Tudományos Konferenciájának tanulmánykötete*, Komárom 2015. pp. 311-318.

Petz, T.: *Relációk, függvények, sorozatok*. In: E. Herendiné Kónya (Eds.): *A matematika tanítása az alsó tagozaton* Budapest: Nemzedékek Tudása Tankönyvkiadó Zrt, 2013. pp. 316-352